Biodiversity and economic growth: Trade-offs between stabilization of the ecological system and preservation of natural dynamics

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Abstract

This paper investigates how optimal economic growth can affect the natural evolution of the ecological system. Policy makers may perform defensive actions to protect biodiversity. These actions, however, may deeply modify the natural ecological dynamics so that the resulting equilibrium has different dynamic features with respect to the equilibrium that would have naturally emerged without human intervention. To investigate this issue more deeply, we analyze the impact that economic activity and environmental defensive choices can have on the natural ecological dynamics depending on whether agents care or do not care for biodiversity. Using an optimal growth model with pollution, in which biodiversity loss may be caused by the negative side-effects of aggregate production, we show that human action can modify the stability of the ecological fixed points. In particular, from the simple analytical formulations adopted in the paper, it emerges that – when the levels of the species is sufficiently low at the fixed point and agents care for biodiversity – human intervention may cause a stabilization of the fixed point and thus avoid the extinction of a species, even in the absence of defensive expenditures specifically finalized at the protection of that species. It follows that the protection of biodiversity (through the stabilization of the ecological system) may come at the cost of a change in the original features of the natural dynamics. Moreover, it is shown that a limit cycle may arise through a Hopf bifurcation from the interaction of the economic and ecological systems even though none of the two systems taken separately admits a limit cycle.

Keywords: Biodiversity; Economic growth; Defensive actions; Ecological dynamics

1. Introduction

In recent years, special attention has been devoted to the loss of biodiversity of the ecosystems, mainly for two reasons. In the first place, because many researchers believe that this loss is currently occurring at an unparalleled rate in human history. Although similar episodes of destruction of biodiversity have occurred in the past (as it was the case with the extinction of the dinosaurs) the present loss of biodiversity appears to be related to human activity and not to exogenous conditions.
natural phenomena (see e.g. Heal, 1994). In the second place, because biodiversity is considered essential for human survival, and in many cases it cannot be replaced by man-made physical capital. These two categories of reasons help to explain why so much attention has been recently devoted to this subject by both politicians and scholars. As to the political debate, at the Johannesburg Summit of 2002 the conservation of biodiversity was recognized as a fundamental prerequisite for the achievement of sustainable development and as one of the absolute priorities of future environmental policies. As to the academic debate, many contributions have sought to make an economic evaluation of biodiversity. Certain authors (see e.g. Montgomery et al., 1999) have proposed a theoretical framework from which the value to be attributed to biodiversity could be derived in order to guide the decisions of the policy makers. Other authors, instead, have criticized the traditional approach of economic theory based on the identification of the correct market price for natural resources, emphasizing the fact that the exchange value of biodiversity is only a tiny portion of its total value (see e.g. Gowdy, 1997) and that neither very high nor very low market prices can ensure the survival of a particular species (McDaniel and Gowdy, 1998). In more general terms, several contributions (see e.g. von Amsberg, 1995) question the intertemporal efficiency of the markets in guiding investment decisions under risk and argue that the excessive reduction of biodiversity can be seen as a specific example of this general result.

Alongside this strand of literature dealing with the market’s capacity to evaluate biodiversity, several authors (see e.g. Perrings, 1995; Atkinson et al., 1999) have focused on the relationship between sustainable development, stability and resilience. The latter are two distinct though related properties of the ecological system. Stability describes the ability of a system to return to an equilibrium after a temporary disturbance, while resilience concerns the ability of a system to absorb changes and maintain its integrity when subject to perturbations like a stress or a shock. Resilience thus determines the persistence of the relationships within a natural system and several authors (e.g. Common and Perrings, 1995) suggest that it can be closely approximated by the biodiversity existing within the ecosystem. Barbier and Schulz (1997) and Li and Löfgren (1998) have investigated how the interaction between anthropic activities and biodiversity may affect the stability of the economic and ecological systems, using optimal control models where the biodiversity level is represented by a single aggregate state variable (a biodiversity index).

We aim to contribute to this literature by examining how economic growth and the defence of biodiversity can alter the stability of the natural ecological dynamics. For this purpose, we will investigate a simple model in which a standard economic growth dynamics is coupled with an ecological dynamics. Differently from Barbier and Schulz (1997) and Li and Löfgren (1998), we will use a disaggregated approach, analyzing an ecological dynamics that describes the interaction between two different species rather than an aggregate biodiversity index.1

The interaction between the economic and the ecological dynamics has been the object of several studies in the fishery literature (e.g. Rosser, 2000; Hommes and Rosser, 2001; Prandham and Chaudhuri, 1999). To the best of our knowledge, however, the contributions in that literature generally look only at the stability of the joint economic–ecological equilibrium rather than at the changes in the stability of the equilibrium as we pass from a purely ecological system to a joint economic–ecological system. Using a model of optimal harvesting, for instance, Prandham and Chaudhuri (1999) show a numerical example in which the optimal fishery equilibrium is a saddle point that can be reached by the economic trajectories but do not perform any analysis to compare it with the stability of the original ecological equilibrium.2

Differently from that literature, moreover, we use here an optimal pollution (rather than optimal harvesting) model, in which economic growth has positive ecological effects through a rise in defensive environmental choices and negative ecological effects through

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1 To the best of our knowledge, the interaction between a standard economic growth dynamics and a disaggregated ecological dynamics has been mainly neglected so far for the analytical problems that arise in this case. In fact, even a simple optimal control model with only three state variables (one for physical capital and two for the ecological species into account) gives origin to six differential equations (three for the state variables and three for the co-state variables) that can hardly be investigated analytically.

2 As it can be easily verified, if we use the same parameter values adopted in Prandham and Chaudhuri (1999) the fixed point of the ecological dynamics can be a saddle point or an attractor. This implies that in their model human intervention can modify the nature of the equilibrium.
an increase in aggregate production. To illustrate the latter effect consider, for instance, the damages provoked to the immune and reproduction systems of several species (e.g. killer whales) by the high levels of PCB and DDT accumulated in the sea due to human production activities. Aggregate production, moreover, may contribute to the observed climate change that is responsible for the sharp reduction or extinction of several animal and vegetal populations. In this sense, in line with several works in the literature (e.g. Brock, 1977; Forster, 1973; Tähvonen and Knuutilainen, 1993), the model can be seen as an optimal growth model with pollution, in which biodiversity loss may be caused by the negative side-effects of the economic activity rather than by human consumption of the species, as it occurs in optimal harvesting models.

The natural law of evolution can obviously lead to the extinction of some species. However, as Barney (1980) has pointed out, most of the recent biodiversity loss is due to the impact of human activity. To counterbalance this trend and avoid the extinction of single species, policy makers can carry out defensive actions. Such defensive actions can be specifically aimed at preserving particular species or generically designed to protect the habitat where species live. For instance, providing to an endangered species the food it needs to survive is an example of specific defensive expenditure, whereas introducing costly policy measures to reduce polluting activities in a biodiversity rich area can be interpreted as a generic defensive expenditure. All defensive expenditures, whether generic or specific, may contribute to defend biodiversity. However, they may also deeply alter the ecological dynamics generated from the natural interaction between the species. As a consequence, human activity can lead to an ecological dynamics that differs from the one that would have naturally emerged without the intervention of mankind. This possibility poses the questions that we will try to investigate in this work, that is: how does human intervention affect the stability of the ecological equilibria? Can generic defensive expenditures be sufficient to avoid biodiversity loss? In the following analysis, we will show that if the equilibrium level of the species taken into account is high enough, no change will occur in the stability features of the equilibrium as we pass from a natural system (without human intervention) to an integrated economic–ecological system (with human intervention). In this case, in fact, the ecological populations are far from extinction, therefore the dynamic features of the ecological equilibrium may be preserved with no risk of biodiversity loss even if the ecological equilibrium is a repellor. If, on the contrary, the equilibrium level of the species is relatively low, human intervention may stabilize the fixed point to ensure the species’ persistence and thus avoid the risk of biodiversity loss even though the defensive actions are not specifically aimed at single species. This poses a possible trade-off between the protection of biodiversity (through the “stabilization” of the ecological system) and the conservation of the intrinsic features of the natural dynamics. Finally, from the analysis of the model we find that the dynamics that arise in the integrated economic–ecological model can be more complex than the ecological and economic dynamics taken separately. In particular, we show that a limit cycle may arise through a Hopf bifurcation in the integrated model although none of the two systems admits a limit cycle on its own.

2. The model

For the sake of simplicity, let us assume that the ecological dynamics is given by the following linear system:

\[
\begin{align*}
\dot{x} &= \beta_1 + \rho_1 x + \gamma_1 y \\
\dot{y} &= \beta_2 + \rho_2 y + \gamma_2 x
\end{align*}
\]  

where \(\beta_1, \beta_2 > 0\) while the other parameters may assume positive or negative value depending on the type of interaction occurring within each species as well as between them. We have deliberately chosen to keep the stylized ecosystem as simple as possible because in this study we will concentrate on the local stability analysis of the fixed points of the ecological dynamics. In this regard, system (1) presents the interesting feature that...
Expenditure, whereas introducing costly policy measures to reduce species the food it needs to survive is an example of specific defensive habitat where species live. For instance, providing to an endangered species5; in particular, we assume that ecological defensive expenditures can be specifically aimed at preserving particular species or genetically designed to protect the habitat where species live. For instance, providing to an endangered species the food it needs to survive is an example of specific defensive expenditure, whereas introducing costly policy measures to reduce

cal dynamics, augmented with the (positive and negative) effects of economic activity, is described by the system:

\[ \begin{align*}
    \dot{x} &= \beta_1 + \rho_1 x + \gamma_1 y - \epsilon_1 k^\alpha + \eta_1 d^\alpha \\
    \dot{y} &= \beta_2 + \rho_2 y + \gamma_2 x - \epsilon_2 k^\alpha + \eta_2 d^\alpha
\end{align*} \]  

(4)

(5)

where \( \sigma_1, \sigma_2, \epsilon_1 \) and \( \epsilon_2 \) are strictly positive parameters; \( \epsilon_1 \) and \( \epsilon_2 \) measure the negative impact of aggregate production on the dynamics of the two species \( x \) and \( y \), respectively. The parameter \( \mu \) is such that \( 0 < \mu < 1 \); this implies that the positive effect on the species evolution of an additional unit of defensive expenditure decreases when \( d \) increases.

Let us assume that there is a representative agent in the economy whose welfare depends, in each instant of time \( t \), on the consumption of the produced good \( c(t) \) and on the amount of the two species \( x(t) \) and \( y(t) \). Following Li and L"ofgren (1998), it can be argued that \( x(t) \) and \( y(t) \) enter the representative agent’s utility function both for their amenity values and for their optional uses in the future. In particular, we will analyze dynamics under the following two alternative utility function specifications:

\[ U(x, y, c) = q_1 x + q_2 y + q \ln c \]  

(6)

\[ U(x, y, c) = q_1 \ln x + q_2 \ln y + q \ln c \]  

(7)

where \( q_1, q_2 \) and \( q \) are strictly positive parameters. In (7), we assume a logarithmic utility function in each argument; under such a function, strictly positive values of \( x, y \) and \( c \) are essential for representative agent’s welfare; more specifically, if coerteris paribus \( x, y \) or \( c \) goes to zero, the partial derivatives of (7) with respect to \( x, y \) and \( c \) (respectively) go to \(+\infty\) and the representative agent suffers an infinite welfare loss. In the utility function (6), on the contrary, only consumption is considered as essential by the representative agent, whereas this no longer applies to the species \( x, y, c \); so that in this case nothing prevents single species from extinction. Differently from function (7) in which the agents care for biodiversity, the utility function (6) rep-

\[ J = \begin{pmatrix} \rho_1 & \gamma_1 \\ \rho_2 & \rho_2 \end{pmatrix} \]

and the characteristic polynomial is:

\[ z^2 - (\rho_1 + \rho_2) z + \rho_1 \rho_2 - \gamma_1 \gamma_2 \]  

(2)

Therefore, the fixed point is a saddle point if \( \rho_1 \rho_2 - \gamma_1 \gamma_2 < 0 \), is an attractor if \( \rho_1 \rho_2 - \gamma_1 \gamma_2 > 0 \) and \( \rho_1 + \rho_2 < 0 \) and is a repellor if \( \rho_1 \rho_2 - \gamma_1 \gamma_2 > 0 \) and \( \rho_1 + \rho_2 > 0 \).

The dynamics (1) can be thought of as the evolution that species would naturally follow if we let Nature take its course and will be our benchmark throughout the paper. The objective of our study is to analyze how economic growth and the underlying agents decisions can modify the stability characteristics of the fixed point of the ecological dynamics (1). For this purpose, we will examine a simple growth model that links the ecological dynamics (1) to a capital accumulation dynamics. In particular, we suppose there is a single produced good in the economy which is produced by capital alone (\( k \)). The corresponding output can be used for capital accumulation, consumption (\( c \)) and defensive expenditures (\( d \)); therefore, capital accumulation is described by the following differential equation:

\[ \dot{k} = k^\alpha - c - d \]  

(3)

where \( k^\alpha \) is the production function of the economy; the parameter \( \alpha \) is such that \( 0 < \alpha < 1 \).

Economic activity influences the natural ecological dynamics through the negative effects generated by the pollution due to aggregate production \( k^\alpha \) and through the positive effects of environmental defensive expenditures. In our model we assume a defensive technology which prevents discrimination between the two species5; in particular, we assume that ecological defensive expenditures can be specifically aimed at preserving particular species or genetically designed to protect the habitat where species live. For instance, providing to an endangered species the food it needs to survive is an example of specific defensive expenditure, whereas introducing costly policy measures to reduce polluting activities in a biodiversity rich area can be interpreted as a generic defensive expenditure.
The representative agent chooses consumption and defensive expenditures so as to maximize  the present discounted value of the utility function subject to the physical capital accumulation dynamics and the “modified” ecological dynamics, that is, the natural ecological dynamics (1) as modified by human intervention. Thus, we obtain the following optimal control problem:

\[ \max_{c,d} \int_0^\infty U(x, y, c)e^{-\rho t} dt \]

subject to equations (3)–(5) (8)

where the parameter \( r \) indicates the subjective discount rate \( (r > 0) \).

The current value Hamiltonian function associated to the optimization problem (8) is:

\[ H = U(x, y, c) + \lambda(k^e - c - d) \]

\[ + \phi_1 (r_1 x + y + \gamma_1 y - \gamma_1 k^e + \sigma_1 d^e) \]

\[ + \phi_2 (r_2 y + y x - \gamma_2 k^e + \sigma_2 d^e) \]

where \( \lambda, \phi_1 \) and \( \phi_2 \) are, respectively, the multipliers of \( k, x \) and \( y \), and have the usual interpretation of "shadow prices".

From the maximum principle, the first-order necessary conditions for optimality are:

\[ \frac{\partial H}{\partial c} = U_x - \lambda = 0 \] (9)

\[ \frac{\partial H}{\partial d} = -\lambda + \mu \sigma_1 d^{e-1} \phi_1 + \mu \sigma_2 d^{e-1} \phi_2 = 0 \] (10)

\[ x = \frac{\partial H}{\partial \phi_1} = \phi_1 + r_1 x + y + \gamma_1 y - \gamma_1 k^e + \sigma_1 d^e \] (11)

\[ y = \frac{\partial H}{\partial \phi_2} = \phi_2 + r_2 y + y x - \gamma_2 k^e + \sigma_2 d^e \] (12)

\[ k = \frac{\partial H}{\partial \lambda} = k^e - c - d \] (13)

\[ \theta = e^{\rho t} \frac{\partial H}{\partial \lambda} = (r - \rho_1) \theta - \gamma_2 \eta - U_x \] (14)

\[ \eta = e^{\rho t} \frac{\partial H}{\partial \eta} = (r - \rho_2) \eta - \gamma \theta - U_y \] (15)

\[ \lambda = e^{\rho t} \frac{\partial H}{\partial \lambda} = \lambda(r - \alpha k^{e-1}) + (b k_1 + \sigma_2 k^{e-1}) \] (16)

Substituting the optimal choices of the control variables \( c \) and \( d \) in the equations (11)–(16), we obtain a dynamic system in the space \( (x, y, k, \theta, \eta, \lambda) \). Given the initial values of the state variables \( x, y \) and \( k \), the initial values of multipliers are fixed according to the transversality conditions \( \lim_{t \to +\infty} x(t) \cdot \theta(t) \cdot e^{-\rho t} = \lim_{t \to +\infty} y(t) \cdot \eta(t) \cdot e^{-\rho t} = \lim_{t \to +\infty} k(t) \cdot \lambda(t) \cdot e^{-\rho t} = 0 \). Note that such conditions are satisfied by every trajectory of system (11)–(16) approaching a fixed point or a limit cycle.

3. Stability analysis when agents do not care for biodiversity

3.1. Basic mathematical results

Before analyzing the local stability of the fixed points of dynamics, observe that if the Jacobian matrix of system (11)–(16), evaluated at a fixed point, admits three eigenvalues with strictly negative real parts (and hence three eigenvalues with positive real parts), then the fixed point can be achieved: namely, for any initial state \((x_0, y_0, k_0)\) sufficiently close to the fixed point values of \((x, y, k)\), it is possible to determine the initial values of the multipliers \((\theta_0, \eta_0, \lambda_0)\) such that the economic growth trajectory eventually converges to the fixed point. If the number of eigenvalues with negative real parts is strictly less than three, this is no longer possible a part for a negligible set of initial states \((x_0, y_0, k_0)\). Therefore, if the fixed point of system (1) is an attractor, then the stability properties of the natural dynamics will be preserved if the fixed point of system (11)–(16) has three or more negative eigenvalues. Similarly, if the fixed point of system (1) is a repellor or a saddle point, its stability properties will be preserved with human intervention provided the fixed point of system (11)–(16) has less than three negative eigenvalues.
In this subsection, we analyze the stability of the integrated system assuming the utility function \( (6) \), namely that agents do not care for biodiversity (i.e. they do not consider positive values of \( x \) and \( y \) as essential in their utility function).

By substituting \( \frac{dx}{dt} = \frac{1}{x} \) and \( \frac{dy}{dt} = q_1 \) in the equations (9), (14) and (15), the Jacobian matrix of system (11)–(16), evaluated at a fixed point \( P^* = (x^*, y^*, k^*, \theta^*, \eta^*, \lambda^*) \), becomes:

\[
J^* = \begin{pmatrix}
\rho_1 & \gamma_1 & -c_1\epsilon(k^*)^{p-1} & H_{k,\theta} & H_{k,\eta} & H_{k,\lambda} \\
\gamma_2 & \rho_2 & -c_2\epsilon(k^*)^{p-1} & H_{\theta,\theta} & H_{\theta,\eta} & H_{\theta,\lambda} \\
0 & 0 & \alpha(k^*)^{p-1} & H_{\eta,\theta} & H_{\eta,\eta} & H_{\eta,\lambda} \\
0 & 0 & 0 & r & -\gamma_2 & 0 \\
0 & 0 & 0 & -\gamma_1 & r & -\rho_2 \\
0 & 0 & -H_{\lambda,\lambda} & \alpha(k^*)^{p-1}\epsilon_1 & \alpha(k^*)^{p-1}\epsilon_2 & 0
\end{pmatrix}
\]

where:

\[
H_{k,\theta} = \frac{\sigma_1}{2\lambda^*}; \quad H_{k,\eta} = \frac{\sigma_1}{2\lambda^*} \\
H_{k,\lambda} = -\frac{\sigma_1\psi}{2\lambda^*}; \quad H_{\theta,\theta} = \frac{\sigma_1}{2\lambda^*} \\
H_{\eta,\eta} = \frac{\sigma_1}{2\lambda^*}; \quad H_{\theta,\eta} = \frac{q}{(\lambda^*)^2} + \frac{\psi^2}{(2\lambda^*)^2} \\
H_{\lambda,\lambda} = -\alpha(a - 1)(k^*)^{p-2}(\lambda - \epsilon_1\theta^* - \epsilon_2y^*)
\]

and \( \psi := \sigma_1\theta^* + \sigma_2\rho^* \).

Let us now analyze the characteristic polynomial of \( J^* \). After some Gauss transformations, we obtain the following characteristic equation:

\[
EQ_3(z) : EQ_2(z) \cdot EQ_1(z) = 0
\]

where

\[
EQ_3(z) := z^2 - (\rho_1 + \rho_2)z + \rho_1\rho_2 - \gamma_1\gamma_2 \\
EQ_2(z) := z^2 + (\rho_1 + \rho_2 - 2\gamma_1)z + \rho_1\rho_2 \\
-\gamma_1\gamma_2 - r(\rho_1 + \rho_2) + \rho^2
\]

Notice that \( H_{ij} \) (\( i, j = 0, 1, \lambda \)) indicates the second partial derivative of the Hamiltonian in correspondence of the fixed point.

Notice that \( EQ_1(z) \) coincides with the characteristic polynomial (2) of the natural dynamics (1). Furthermore, note that, being \( H_{\lambda,\lambda} < 0, H_{\theta,\theta} > 0 \) and \( r = \alpha(k^*)^{p-1} < 0 \) (see (16)), the equation \( EQ_3 \) has two real solutions of opposite signs. We can distinguish three cases depending on whether the fixed point of the dynamics (1) is a repellor, an attractor or a saddle point.

3.2. Case 1: Repelling fixed point for the natural dynamics

The equation \( EQ_3 \) produces two positive eigenvalues with positive real part.

The equation \( EQ_2 \) generates two real positive eigenvalues, having the known term always positive and the coefficient of \( z \) always negative. Finally, the equation \( EQ_1 \) features a negative solution and a positive one. To sum up, the representation of the sign of the real parts of the eigenvalues of \( J \) is the following:

\[
EQ_3 \quad + \quad EQ_2 \quad + \quad EQ_1 \quad +
\]

From the above representation, we observe that there is only one negative eigenvalue; this means that the fixed point has a one-dimensional stable manifold and consequently it cannot (generically) be reached by the economy.

Therefore, the property of instability of the fixed point of the natural dynamics is transferred unaltered to the economic growth dynamics.
3.3. Case 2: Attracting fixed point for the natural dynamics

The equation EQ1 = 0 generates two eigenvalues with negative real part so that the fixed point of the natural dynamics is a sink. The equation EQ2 = 0 gives two positive real eigenvalues, and the equation EQ3 = 0 gives two eigenvalues of opposite signs. In synthesis, the following picture emerges:

EQ1: --
EQ2: ++
EQ3: --

So, we have three negative eigenvalues and the fixed point has a three-dimensional stable manifold; consequently, the fixed point will be reached by the economy if the initial values of $x$, $y$, and $k$ are near enough to the fixed point values $x^*$, $y^*$ and $k^*$.

3.4. Case 3: Saddle point for the natural dynamics

Proceeding as above, it is easy to observe that in this case the configuration of the signs of the eigenvalues is given by:

EQ1: + -
EQ2: + +
EQ3: + -

The negative eigenvalues are two and the stable manifold of the fixed point is two-dimensional. Consequently, in this case the fixed point cannot be generically reached by the growth trajectory.

The analysis developed in this section shows that, whatever the values of $x$ and $y$ at the fixed point of the equations system (11)-(16), the economic activity and the defensive expenditures do not alter the stability properties of the fixed point of the natural dynamics (1). In particular, starting from a position that is sufficiently close to the fixed point of the system (11)-(16), the fixed point will be reached if and only if the natural dynamics (1) has an attracting fixed point. If, on the contrary, the equilibrium of the natural dynamics (1) is a repellor or a saddle point, then the fixed point of the integrated economic–ecological system (11)-(16) will not generically be reached.

3.5. Some numerical examples

Let us assume that the values of parameters of the natural dynamics (1) are as follows:

$\beta_1 = 0.2, \quad \beta_2 = 0.1, \quad \rho_1 = 0.01, \quad \rho_2 = -0.008, \quad \gamma_1 = -0.011, \quad \gamma_2 = 0.008$

The coordinates of the corresponding fixed point turn out to be $\bar{x} = 62.5$ and $\bar{y} = 75$, while the eigenvalues are $\lambda_1, \lambda_2 = 0.001 \pm 0.002645i$. Consequently, the fixed point $(x, y) = (62.5, 75)$ is repelling under the natural dynamics. For the dynamics (11)-(16), we use the following additional parameters specifications:

$a = \mu = 0.5, \quad q_1 = q_2 = 1, \quad q = 10, \quad e_1 = e_2 = 1, \quad \sigma_1 = \sigma_2 = 0.1, \quad r = 0.05$

At the fixed point of the dynamics (11)-(16), the state and control variables take the following values:

$x^* = 154.24, \quad y^* = 136.162, \quad k^* = 0.0622, \quad c^* = 0.0427, \quad d^* = 0.00237$

and the eigenvalues of the Jacobian matrix $J$ are:

$\lambda_1, \lambda_2 = 0.001 \pm 0.002645i, \quad \lambda_3, \lambda_4 = 0.049 \pm 0.002645i, \quad \lambda_5 = -1.979, \lambda_6 = 2.03$

As emerges from this numerical example, although the two populations increase at the fixed point, the instability of the equilibrium of the natural dynamics is transferred to the equilibrium of the integrated economic–ecological system (11)-(16). The fixed point of this system, in fact, has only one eigenvalue with negative real part; therefore, there exists a one-dimensional stable manifold. This implies that, for a given set of initial values of $x$, $y$, and $k$, the fixed point cannot (generically) be reached by choosing the initial values of the co-state variables $\lambda$, $\theta$ and $\eta$.

Fig. 1 shows how a change in $e_1$ (the negative impact of aggregate production on $x$) modifies the equilibrium values of the state variables ($x$, $y$ and $k$), of consumption and of the defensive expenditures. Notice that $x^* = 0$ and $y^* = 0$ when $e_1$ is about 1.70.
so that both species disappear. As a matter of fact, if the negative impact of aggregate production on the preys causes a reduction in their number, this reduces also the number of predators that depend on x for their survival, so that it may eventually lead to the extinction of both species. Consistently with the analysis of the Jacobian matrix, we find that the fixed point can never be reached even when \( x^* \) and \( y^* \) approach zero.

4. Stability analysis when agents care for biodiversity

4.1. Basic mathematical results

Let us now consider the case in which agents care for biodiversity, that is, they suffer an infinite welfare loss from species’ extinction as it occurs with the utility function (7). Plugging \( \frac{\partial U}{\partial c} = q_c, \frac{\partial U}{\partial x} = q_{1x}, \frac{\partial U}{\partial y} = q_{2y} \) in equations (9), (14) and (15), we obtain the Jacobian matrix:

\[
J' = \begin{pmatrix}
\rho_1 & \gamma_1 & -\epsilon_1 (k^y)^{\rho-1} & 0 & 0 & 0 \\
\gamma_2 & \rho_2 & -\epsilon_2 (k^y)^{\rho-1} & 0 & 0 & 0 \\
0 & 0 & a (k^x)^{\rho-1} & H_{0,0} & H_{0,1} & H_{0,2} \\
0 & 0 & a (k^x)^{\rho-1} & H_{1,0} & H_{1,1} & H_{1,2} \\
0 & 0 & a (k^y)^{\rho-1} & H_{2,0} & H_{2,1} & H_{2,2} \\
0 & 0 & a (k^y)^{\rho-1} & H_{k,k} & H_{k,k} & H_{k,k}
\end{pmatrix}
\]
Observe that if the values of $x$ and $y$ are high enough at the fixed point, the $q_1^2(x^*)^2$ and $q_2^2(y^*)^2$ terms in the matrix $J'$ assume values close to zero and are therefore negligible; furthermore, note that, plugging $q_1^2(x^*)^2 = 0$ and $q_2^2(y^*)^2 = 0$ in matrix $J$, then matrix $J'$ is obtained.

This implies that, if $x$ and $y$ are high enough at the fixed point, the stability properties of fixed points with the utility functions (6) and (7) are the same: That is, if both species are sufficiently abundant in correspondence of the fixed point, the fixed point of system (11)–(16) can be (generically) reached if and only if the fixed point of system (1) is an attractor.

So far we have examined the case where $x$ and $y$ are sufficiently high at the fixed point. But what happens if we consider the more general case in which the two variables $x$ and $y$ can assume any possible (positive) value? In this case the analysis of the Jacobian matrix $J'$ turns out to be cumbersome, so that it is convenient to resort to numerical examples. We therefore proceed with a number of simulations, considering in particular the case in which the fixed point of the natural dynamics (1) is a repellor.

### 4.2. Numerical example 1

Let us start by considering the same parameters values taken in the numerical exercise for the utility function (6). Remember that, for that parameters specification, the fixed point of the natural dynamics (1) is a repellor with coordinates $\bar{x} = 62.5$ and $\bar{y} = 75$. Assuming that the preferences of the representative agent are now given by utility function (7), at the fixed point of dynamics (11)–(16) we have:

\[
\begin{align*}
x^* &= 3764.26, \\
y^* &= 2542.85, \\
k^* &= 97.44, \\
c^* &= 7.36, \\
d^* &= 0.00015
\end{align*}
\]

with eigenvalues:

\[
\begin{align*}
\lambda_1^*, \lambda_2^* &= 0.00097 \pm 0.002634i, \\
\lambda_3^*, \lambda_4^* &= 0.049 \pm 0.002645i, \\
\lambda_5^* &= -0.0314, \\
\lambda_6^* &= 0.0814
\end{align*}
\]

Notice that, with these parameter values, the levels of the two population turn out to be rather high at the integrated ecological-economic fixed point. Moreover, the matrix $J'$ has now only one eigenvalue with negative real part, therefore the fixed point of the integrated system cannot generically be reached. However, differently from what happened with the utility function (6), in what follows we show that in the present case the stability features of the fixed point can change with a change in the parameter values that modifies the levels of $x$ and $y$ at the fixed point.

![Fig. 2. Direction of changes in equilibrium values with an increase in (upper curve) and (lower curve) when agents care for biodiversity.](image-url)
The arrows along the curves in Fig. 2 show the direction of the change in the fixed point values of \( x, y \) and \( k \) as \( e_1 \) (along the bold curve) and \( e_2 \) (along the other curve) increase, keeping all other parameters constant. As the figure shows, the fixed point values of the two populations \( x \) and \( y \) decrease as \( e_1 \) increases and/or \( e_2 \) decreases. This seems consistent with our a priori expectations. As a matter of fact, the parameter values assumed in the present simulations imply a prey–predator inter-relationship between \( x \) and \( y \). Therefore, an increase in \( e_1 \) (the negative impact of the economic activity on the preys) tends to reduce not only the preys but also the predators, as it lowers the food that the latter population has at disposal.

A decrease in \( e_2 \) (the negative impact of the economic activity on the predators) initially increases the number of predators, with a negative effect on the number of preys. The consequent reduction in the level of \( x \), however, tends to decrease the number of predators \( y \), so that – at the end of the day – both populations may turn out to be lower at the integrated fixed point after a reduction in \( e_2 \).

Let us indicate with \( H \) the points where a Hopf bifurcation occurs along the two curves in Fig. 2. The values of the state variables at \( H \) are \((x, y, k) = (1444.58, 305.108, 84.93)\) with \( e_1 = 1.225 \) along the bold curve and \((x, y, k) = (1175.22, 248.808, 84.93)\) with \( e_2 = 0.815 \) along the other curve. Points \( H \) separate the area on the left where the fixed point of the integrated economic–ecological system has only one negative eigenvalue (therefore cannot be reached) from the area on the right in which the fixed point has three negative eigenvalues (so that it can be reached).

As suggested by the signs of these eigenvalues, while the natural ecological dynamics shows a repelling fixed point that cannot therefore be reached by the correspondent trajectories (both eigenvalues being positive), the economic growth trajectories reach the fixed point (in the area on the right and below points \( H \)). A change in the parameter values that reduces \( x \) and \( y \) – as in the cases described above – may move the economy from the left to the right of \( H \) and thus modify the stability features of the fixed point. In particular, if \( x \) and \( y \) get sufficiently low at the fixed point (i.e. below \( H \)), the economic activity and the defensive expenditures produce a "stabilizing" effect that alters the stability properties of the fixed point of the natural dynamics. This seems consistent with our a priori expectations. Since utility is logarithmic in \( x \) and \( y \), agents would suffer an infinite welfare loss from the extinction of one or both species (i.e. the loss of biodiversity). Therefore, when \( x \) and \( y \) are sufficiently low at the fixed point, agents have an incentive to increase their defensive expenditures in order to stabilize the fixed point and thus avoid the biodiversity loss that could derive from oscillations around an equilibrium with low values of \( x \) and \( y \) or from trajectories that move away from that equilibrium. It is interesting to notice that this stabilizing effect exists even though the monitoring activity does not discriminate between the two species. Thus, for example, if a share of income is directed to cleaning up the sea, this may lead to an ecological equilibrium in which both the species \( x \) and \( y \) under consideration (e.g. sharks and sardines) are preserved, although the defensive expenditures for the protection of the sea do not discriminate between the forms of life which populate it.

Finally, notice that, from the Hopf Theorem (see e.g. Medio, 1992), a closed trajectory (limit cycle) is generated when the point \( H \) is crossed moving along the two curves in Fig. 2. In this regard, it seems reasonable to claim that such a limit cycle exists to the left of \( H \), namely when the fixed point cannot be reached by the economic growth trajectories.

This result suggests that complex dynamics may arise in the integrated model even though the economic and ecological components are fairly simple. This seems to confirm what emerges in other contributions in the literature. Using fishery models, for instance, Rosser (2000) and Hommes and Rosser (2001) find that the interaction between the economic and ecological dynamics may generate chaos in the integrated model even though the two original dynamics, taken separately, are very simple. Similarly, Chen (1997) has shown that chaotic dynamics can arise in a globally combined climatic–economic system even
though none of the two systems behaves cahotically on its own.\textsuperscript{11}

4.3. Numerical example 2

The numerical exercise above has shown that agents have an incentive to stabilize the fixed point when \( x \) and \( y \) get sufficiently low at the fixed point. As a matter of fact, in this case oscillations around the fixed point may cause the agents a large welfare loss, given the specification of the utility function (7). Thus, for instance, this is what would happen if the economy runs along a limit cycle in the space \((x, y)\) and the oscillations lead the system very close to the axes.

From the previous results one might conjecture that the economic agents will intervene and stabilize the fixed points whenever \( x \) and \( y \) are sufficiently low at the fixed point. To show that this is not necessarily the case, let us examine the following counterexample. Suppose that the economy is initially on point \( H \) along the bold curve in Fig. 2 and that the intertemporal discount rate \( r \) increases. As Fig. 3 shows, in this case the

\[ \begin{align*}
\text{Fig. 3. Direction of changes in equilibrium values with an increase in the discount rate } r \text{ when agents care for biodiversity.}
\end{align*} \]

fixed point values of \( x, y \) and \( k \) decrease, so that the fixed point moves downwards along the curve. Above point \( H \) in Fig. 3 the fixed point can be reached (as there exist three eigenvalues with negative real part), whereas below \( H \) the fixed point cannot generically be reached (there exists only one negative eigenvalue ). Thus, when the intertemporal discount rate gets sufficiently high, the economy will pass through \( H \) and it seems reasonable to conjecture that the limit cycle generated by the Hopf bifurcation will occur for values of \( x \) and \( y \) below point \( H \). In this case, \( x \) and \( y \) will oscillate around an equilibrium with low levels of the two populations increasing the risk of extinction of one or both species, but no stabilizing intervention will take place despite such a risk.

The mechanism which determines this result may be the following: the lower is the weight that agents give to future well-being, the lower is their level of accumulation and thus also the level of (stabilizing) defensive expenditures which they can afford. If agents do not care sufficiently for the future, therefore, the economic system cannot afford the defensive expenditures that are necessary to stabilize the fixed point and thus avoid the risk of biodiversity loss that could derive from oscillations around an equilibrium with low values of \( x \) and \( y \).

\textsuperscript{11} See also the interesting paper by Rosser (2001) for further discussion and analysis of complex dynamics in economic-ecological systems.
5. Concluding remarks

The objective of the present study was to analyze how human activity can alter the natural ecological dynamics in a model of optimal economic growth where the ecological sector is represented by a very simple dynamics of two interacting species, \( x \) and \( y \). To this end, focusing attention on local stability analysis, we examined the impact which economic growth and defensive expenditures can have on the stability of the fixed point of the ecological dynamics. Although this study is merely a preliminary and not exhaustive analysis of the complex relations existing between economic activity and ecological evolution, some interesting results emerge from the simple analytical formulation adopted here that can contribute to stimulate the current debate on the role of man in the defence of biodiversity.

The analysis of the model suggests that if the agents get constant marginal utility from \( x \) and \( y \) regardless of the two populations’ level, then the stability features of the ecological equilibrium will always be preserved in the integrated economic–ecological system, whatever the level of the two populations. On the contrary, if agents care for biodiversity (that is, if they suffer an infinite welfare loss when each species tends to zero), a stabilization of the fixed point of the ecological dynamics can be observed, even in the absence of defensive expenditures specifically finalized at the protection of that species. Economic agents are incentivized to stabilize a fixed point when \( x \) and \( y \) are sufficiently low. As a matter of fact, in this case the agents have an incentive to make defensive expenditures in order to avoid the loss of biodiversity (i.e. the extinction of one or both species) that could derive from oscillations around an equilibrium with low values of \( x \) and \( y \). Even in this case, however, the stabilization effect may not occur if the agents give little weight to the future, so that they make little defensive expenditures to prevent the risk of biodiversity loss because, for example, they expect such a risk to occur only in a far distant future.

From the above analysis, it follows that a possible trade-off between preservation of biodiversity (through the stabilization of fixed points) and preservation of the natural dynamics may emerge. Such a trade-off can pose policy makers in front of a dilemma if agents want to preserve not only single species but also the nature of the ecological relationships. For instance, when visiting an African park, tourists wish to see a lion hunting a gazelle, since this is regarded as a ‘natural’ though cruel spectacle that shows the ecological dynamics how it would naturally be in the absence of any human intervention. These tourists would not probably be that excited in seeing a lion and a gazelle separately, living on their own, or separately fed by those managing the African park, as this is something they could easily see in a zoo. This seems to suggest that these people want to preserve the prey–predator dynamics that characterize the relationship between the two species. On the other hand, people may also care for biodiversity to be preserved. In other words, they do not want the lions to eat all the gazelles. Therefore, if the lion–gazelle dynamics has a repelling fixed point (that would naturally lead, for instance, to the gazelles extinction), they may want to intervene by stabilizing such a dynamics to ensure that some gazelles may survive. In our opinion, this may exemplify the conflict between stabilization and preservation of the natural dynamics that emerge in the paper.

This conflict suggests some extensions that would be interesting to examine in the future. The tendency to modify the natural dynamics, in fact, can be even more accentuated in reality by policy makers’ choices and

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\[ \text{Notice that the kind of inter-relationship between the two species (measured by parameters } \gamma_1 \text{ and } \gamma_2 \text{ in the model) influences, in its turn, the stability features of the fixed point.} \]

\[ \text{13 We are fully aware that little attention has been given so far to the preservation of the ecological dynamics, so that only a subset of the whole population (e.g. the tourists in the previous example) might care for the nature of the ecological equilibrium. However, one cannot exclude that more and more agents may develop more sophisticated ecological preferences in the future, once they have satisfied their basic needs and their income increase. This is what actually seems to happen to the environmental preferences of the agents, whose demand for environmental quality is highly elastic to their income level (cf. Grossman, 1995). Although we are not aware of any empirical study on this specific issue, several contributions in the vast literature on the so-called environmental Kuznets curve have found empirical support for this hypothesis as far as pollution and other ecological problems are concerned.} \]
species’ behaviors. Policy makers may be more willing to protect from extinction a species that is perceived as a defenceless prey (e.g. the panda) rather than some aggressive and dangerous predator (e.g. the shark). As a consequence, they may finance more the interventions that defend one species than another. Similarly, in the management of a natural park, the managing body may have a specific ecological model in mind (that differs from that of the natural ecological dynamics) in which one species may be preferred to another because it attracts more tourists to the park. In this case, the decision makers may be induced to stabilize the fixed point at a higher number of individuals of one species than another. In both examples, therefore, human preferences may help to preserve single species from extinction, but they also deeply modify the natural interaction dynamics among different species.

Moreover, defensive expenditures may alter the natural ecological dynamics even because they change species’ behavior. Suppose, for instance, that $x$ and $y$ represent, respectively, the number of sardines and of tuna fish, and that the defensive intervention takes the form of supplying man-made food to the two species. In this case, the sardines and tuna fish would increase in number, but the tuna fish would partially lose their typical predatory instinct towards the sardines, thus altering what would have been the natural evolution of the two species. Feeding species to avoid their extinction can therefore preserve biodiversity, but this may come at the cost of an irreversible loss in the dynamics of the ecosystem.

In conclusion, biodiversity loss is a serious threat for mankind. As a matter of fact, biodiversity supports both human survival (favoring the ecological equilibrium) and human activities (providing, for example, active principles which can be exploited for the production of medicinal products in the pharmaceutical industry). Policy makers, therefore, should protect biodiversity to avoid potentially irreversible damages to the ecological systems. However, as shown in this study, the effort to protect biodiversity may modify the ecological dynamics that would naturally arise from the interaction between species. This may hinder our capacity to preserve at the same time biodiversity and the original features of the natural dynamics, posing the policy maker in front of a dilemma of choice between these two conflicting goals. In our opinion, further research should be devoted in the future to such a complex argument that has been generally neglected so far in the debate on the protection of biodiversity.

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